How Rare are Fancy Serial Numbers? By Dave Undis

OTES WITH FANCY SERIAL NUMBERS COMMAND a premium from some currency collectors. A solid serial number, a radar, a ladder, or several other types of fancy serial numbers can make a note worth much more than other notes with the same denomination, from the same series, and in the same condition. Just how rare are these fancy serial numbers?

Modern United States currency features eight-digit serial numbers from 00000001 to 99999999. There are 99,999,999 different eight-digit serial numbers. To keep things simple, we'll assume that all serial numbers are equally likely to appear on a randomly selected note. In reality, some serial numbers are harder to find because the Bureau of Engraving & Printing usually removes them from production before circulation, replacing them with star notes. Also, print runs don't always start at 00000001 or go all the way up to 99999999. But this doesn't really affect our analysis much. (To make some of the arithmetic below simpler, we'll sometimes include the non-existent 00000000 serial number, making a total of 100,000,000 serial numbers, but we'll always remember to remove that non-existent note from our totals.)



In a solid serial number, every digit is the same. There are nine solid serial numbers: 11111111, 22222222, 33333333, 44444444, 55555555, 666666666, 77777777, 888888888, and 999999999.



In a ladder serial number, each digit is one number higher (or lower) than the previous digit. There are six eight-digit ladder serial numbers: 01234567, 12345678, 23456789, 76543210, 87654321, and 98765432. (There are eight if you also count 34567890 and 09876543.)



In a radar serial number, the second four digits are the reverse of the first four digits. Examples are 11188111 and 20066002. In a repeater serial number, the second four digits are a repeat of the first four digits. Examples are 11181118 and 20062006.

In calculating the number of radars and repeaters, it's useful to divide the eight-digit serial number into its first and second halves. Consider the first half. There are 10,000 possible numbers: 0000 through 9999. Now consider the second half. Again, there are 10,000 possible numbers. Of these 10,000, only one will be the same as the first half and only one will be the reverse of the first half. So there are 10,000 repeaters and 10,000 radars. These totals include the non-existent 00000000 number and the nine solids. Excluding these, there are 9,990 radars and 9,990 repeaters.



A super radar is a special kind of radar. A super radar is a radar in which the middle six digits are all the same. Examples are 71111117 and 98888889.

There are 99 super radars: 01111110, 02222220, 03333330 ... 97777779, 98888889, and 99999999. Nine of these are solids. Excluding these, there are 90 super radars.



A super repeater is a special kind of repeater. A super repeater is a repeater in which the first two digits are repeated four times. Examples are 26262626 and 78787878.

There are 99 super repeaters: 01010101, 02020202, 03030303 ... 97979797, 98989898, and 99999999. Nine of these are solids. Excluding these, there are 90 super repeaters.



A radar repeater is both a radar and a repeater. Examples are 26622662 and 87788778. There are 99 radar repeaters: 01100110, 02200220, 03300330 ... 97799779, 98899889, and 99999999. Nine of these are solids. Excluding these, there are 90 radar repeaters.



In a double quad serial number, the first four digits are all the same and the last four digits are all the same, but a different number. Examples are 11114444 and 222266666. There are 99 double quads: 00001111, 00002222, 00003333 ... 99997777, 99998888, and 999999999. Nine of these are solids. Excluding these, there are 90 double quads.



In a seven-in-a-row serial number, seven digits in a row are all the same. Examples are 11111112 and 35555555. For each of the nine solid serial numbers, you can make 18 seven-in-a-row serial numbers: nine by changing the first digit and nine by changing the last digit. For example, the nine seven-in-a-row serial numbers you can make by changing the first digit of solid serial number 99999999 are 09999999, 19999999, 29999999, 39999999, 49999999, 59999999, 69999999, 79999999, and 89999999. You can also make 18 seven-in-a-row serial numbers by changing the first or last digit of the non-existent 00000000 serial number. So there are 180 seven-in-a-row notes.



In a seven-of-a-kind serial number, seven of the digits are all the same. Examples are 44444424 and 99899999. For each of the nine solid serial numbers, you can make 72 seven-of-a-kind serial numbers: nine by changing each of the eight digits. You can also make 72 seven-of-a-kind serial numbers by changing each of the eight digits of the non-existent 00000000 serial number. So there are 720 seven-of-a-kind notes. This includes the 180 seven-in-a-row notes.



In a binary serial number, only two numbers appear in the eight digits. Examples are 10100111 and 11181888. To see how many binary serial numbers there are, pick two different numbers from zero through nine. 20% of all serial

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numbers have one of these two numbers as the first digit. Of these, 20% have one of these two numbers as the second digit. Of these, 20% have one of these numbers as the third digit. Continue similarly through the eighth digit. So $20\% \times 20\% \times 2$

The table below shows how many different fancy serial numbers appear in a run of 99,999,999 notes. It also shows the odds of seeing each type of fancy serial number on a randomly selected note from a run of 99,999,999 consecutive notes.

Fancy serial number type	Number of occurrences in a 99,999,999 note run	Odds that a random note will be this type
Solid	9	1 in 11,111,111
Ladder	6	1 in 16,666,667
Radar	9,990	1 in 10,010
Repeater	9,990	1 in 10,010
Super radar	90	1 in 1,111,111
Super repeater	90	1 in 1,111,111
Radar repeater	90	1 in 1,111,111
Double quad	90	1 in 1,111,111
Seven-in-a-row	180	1 in 555,556
Seven-of-a-kind	720	1 in 138,889
Binary	11,430	1 in 8,749
-100-100-50		

As previously mentioned, a random note will typically not come from a run of 99,999,999 notes. But this table still gives a good starting point for further analysis and collecting enjoyment.

